METRIC LEARNING · LATENT EXPLORATION · MEDICAL IMAGING

Finsler-Randers Metric Learning for *Direction-Aware* Latent Space Interpolation

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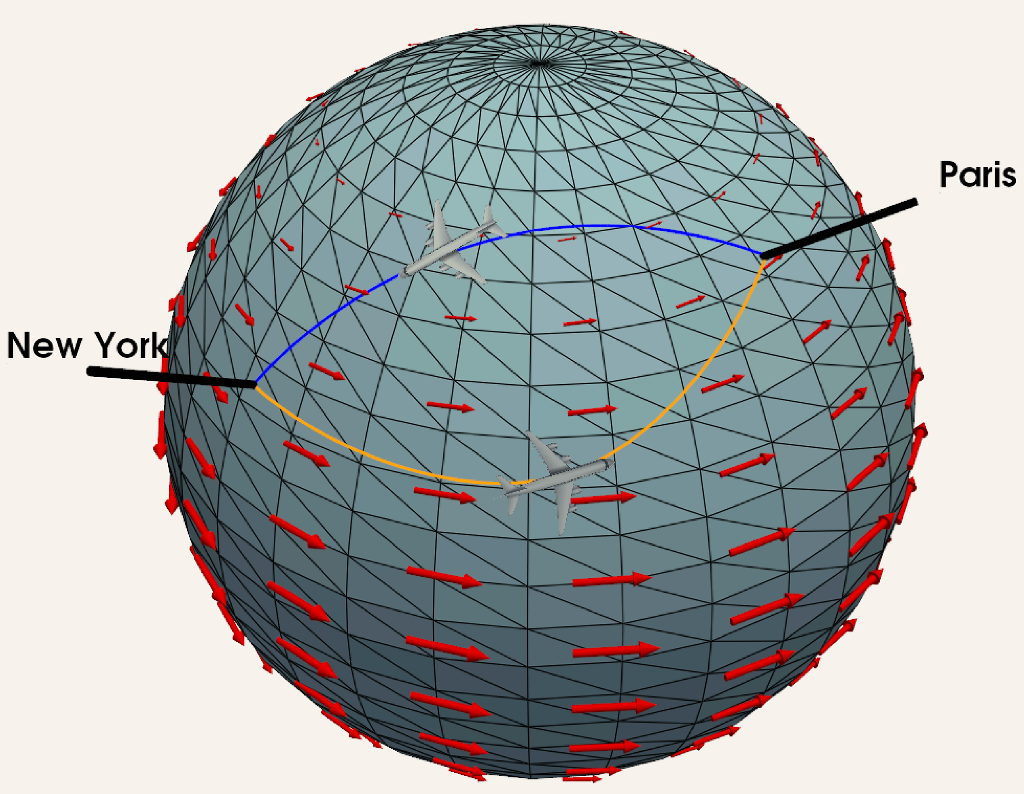
Standard latent space interpolation assumes symmetric distances — but real processes like disease progression are inherently directional. We introduce a **Finsler-Randers metric learning** framework that adds a learned vector field to a Riemannian base metric, encoding directional bias at negligible computational cost. Geodesics under our metric respect monotonic sequential constraints where all symmetric methods fail.

01 MOTIVATION

Why Symmetry Fails ?

Usual Riemannian geometry (Euclidean included) uses **symmetric distances**: $\mathrm{dist}(a,b) = \mathrm{dist}(b,a)$. But real world processes are often irreversible, e.g. Alzheimer's progression, tumor growth... Such interpolated geodesics can produce **non-monotone trajectories that are clinically impossible**.



> *"A Paris→NY flight takes longer than NY→Paris because of the Gulf Stream — computing the geodesic on a sphere is not sufficient. We need to model asymetry !"*

02 BACKGROUND

A primer on Riemannian Geometry

A **Riemannian metric** G equips the latent space with a smooth inner product. This allows to define the local energy as $\|v\|^2_{G(z)} = v^\top G(z) v$

For a VAE with encoder $f(x_i) = (z_i, \Sigma_i)$ and decoder $g(z_i) \approx x_i$, we consider two metrics:

VAE COMETRIC

$$G^{-1}(z) = \sum_i \exp\left(-\frac{\|z - z_i\|_{\Sigma_i^{-1}}}{2\tau^2}\right) \Sigma_i^{-1} + \lambda \mathrm{Id}$$

PULLBACK COMETRIC

$$G_{\mathrm{pb}}(z_i) = J_f(z_i)^\top J_f(z_i)$$

$$G_{\mathrm{pb}}^{-1}(z) = \sum_i \exp\left(-\frac{\|z - z_i\|_{G_{\mathrm{pb}}^{-1}(z_i)}}{2\tau^2}\right) G_{\mathrm{pb}}^{-1}(z_i) + \lambda \mathrm{Id}$$

03 OUR METHOD

Randers Metric Learning

A **Randers metric** is the minimal asymmetric extension of a Riemannian norm. It adds a 1-form (vector field) ω to encode a preferred direction of travel. Note that $F(\cdot, v) \neq F(\cdot, -v)$.

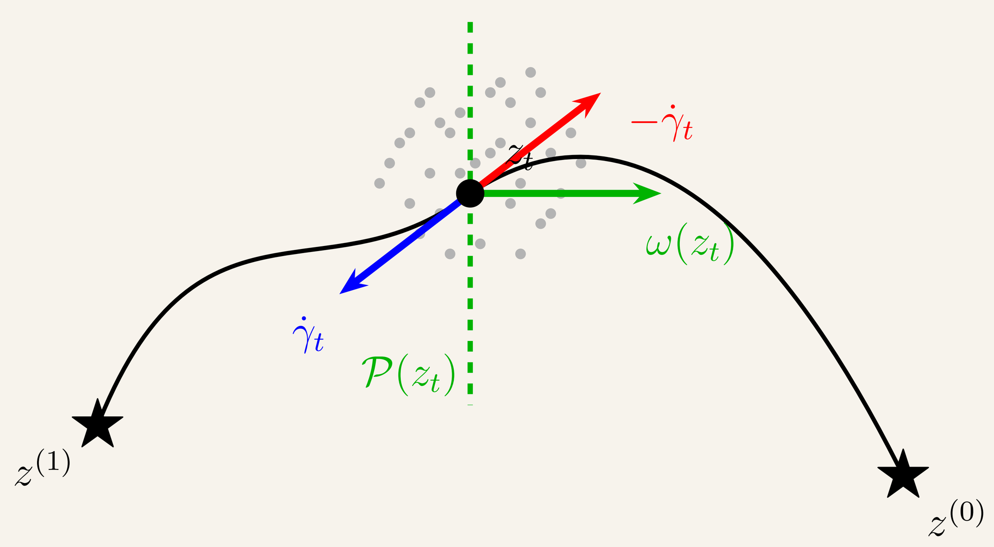
RANDERS METRIC

$$\mathcal{F}(z, v) = \sqrt{v^\top G(z) v} + \langle \omega(z), v \rangle$$

SEQUENTIAL HYPERPLANE

$$\mathcal{P}(z) = \{v \in T_z\mathcal{Z} : \langle \omega(z), v \rangle = 0\}$$

Trajectories *antipodal* to ω have lower energy. Hence Randers geodesics tend to cross the **sequential hyperplane** in the correct direction, respecting monotonicity constraints.



04 LEARNING

SVM-Inspired Orientation Learning

We learn $\omega(z) = \rho(z)\phi(z)$ where ϕ indicates the hyperplane normal direction and ρ the strength of the regularization of the Riemannian metric.

LOCAL HINGE LOSS

$$l_\phi(z_i) = \sum_{j \in \mathrm{KNN}(z_i)} \max\left(0, 1 - y_i^j \times \phi(z_i)^\top (z_j - z_i)\right)$$

LOCAL ACCURACY LOSS

$$\mathrm{s\_dst}(z_i, z_j) = y_i^j \times \phi_\theta(z_i)^\top (z_j - z_i)$$

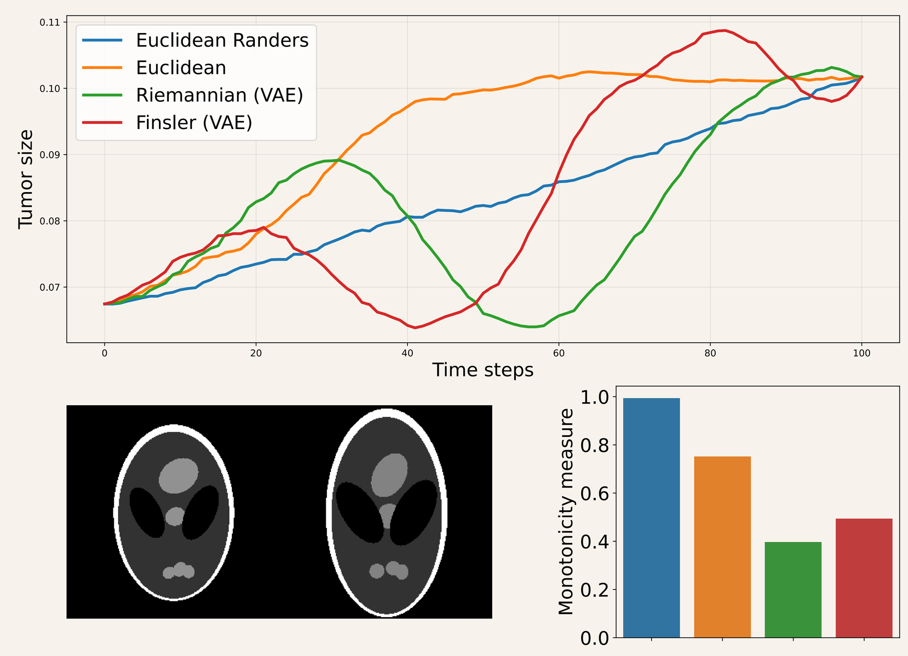
$$\mathrm{ACC}_\phi(z_i) = \frac{1}{K_\phi} \# \{\mathrm{s\_dst}(z_i, z_j) > 0 \mid z_j \in \mathrm{KNN}(z_i)\}$$

$$l_\rho(z_i) = \left\| \rho_\theta(z_i) - 2 \times \max\left(0, \mathrm{ACC}_\phi(z_i) - \frac{1}{2}\right) \right\|^2$$

05 EXPERIMENTS

Results on Shepp-Logan Dataset

Evaluated on a medical imaging phantom (N=1000 train / 500 test) we model tumor size evolution. We compute geodesics between 'scans' with small tumor and large tumor, and evaluate monotonicity of the tumor size along the trajectory. We use monotonicity measure $m \in [0,1]$; higher is better.



MONOTONICITY RESULTS · LATENT DIM = 1024

Method	Score
Euclidean Randers ★	0.84
Euclidean	0.66
Riemannian (VAE)	0.56
Finsler (VAE)	0.48
Riemannian (Pullback)	0.58
Finsler (Pullback)	0.50

06 CONCLUSION

Asymmetry, Not Curvature

A lightweight directional bias on a simple Euclidean base outperforms elaborate Riemannian metrics.
Futur work : Scale to real-world dataset, experiment with geodesic extrapolation, & add a statistical framework to enable sampling of longitudinal trajectories.

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